Enrollment No: _______________________ **Exam Seat No:**____________________

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Linear Algebra-II

Subject Code: 4SC04LIA1 **Branch: B.Sc. (Mathematics)**

Semester: 4 **Date: 05/05/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) An angle between vectors $u = (1,0)$ and $v = (0,1)$ is______. **(01)**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) π (d) 0

b) If $u = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ then $\|u\|$ =______. **(01)**

(a) 1 (b) 2 (c) 3 (d) 4

c) The vectors u and v are orthogonal to each other in inner product space V, then $< u, v > =$ ________. **(01)**

(a) 0 (b) 1 (c) -1 (d) π

d) The Eigen value of matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ are _______. **(01)**

(a) 1 and -2 (b) 1 and 2 (c) -1 and 2 (d) None

e) An angle between $v = (x, y)$ and $w = (-y, x)$ where $x \neq y \neq 0$. **(01)**

(a) 0 (b) π (c) 2π (d) $\frac{\pi}{2}$

f) Is set $S = \{u_1, u_2\}$ where $u_1 = (2,0)$ and $v_2 = (0,2)$is basis for R^2? **(01)**

(a) True (b) false (c) both (d) none

g) Define : Angle between vectors in inner product space V. **(01)**

h) Define: Orthogonal set in inner product space V. **(01)**

i) Define : Inner product space. **(02)**

j) Define : Orthonormal basis for inner product space V. **(02)**

k) Define : Orthogonal compliment. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Compute the angle between $u = e_1$ and $v = e_2 + e_2$ in R^2, where $e_1 = (1,0)$ and $e_2 = (0,1)$.	**(05)**
	b)	Let $x, y, z \in V$ let $x \perp y$ and $y \perp z$ then prove that $x \perp (\alpha y + \beta z$, $\forall \alpha, \beta \in F$.	**(05)**
	c)	Prove that $\|x\| = \|y\| \Leftrightarrow (x - y) \perp (x + y)$.	**(04)**
Q-3		**Attempt all questions**	**(14)**
	a)	Let v and w are two non-zero vectors in V with $v \perp w$ let $\alpha, \beta \in F$ such that $\alpha v + \beta w = 0$ then $\alpha = \beta = 0$.	**(05)**
	b)	Prove that for any $x, y \in V$, show that $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$.	**(05)**
	c)	Show that the vectors $u_1 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$, $u_2 = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$, $u_3 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ are orthogonal to each other in inner product space R^3.	**(04)**
Q-4		**Attempt all questions**	**(14)**
	a)	let S be a non-empty subset of V, let $S = \{v \in V \mid < v, s >= 0 \ , \forall s \in S\}$ then prove that $S^{\perp}$ is vector subspace of V.	**(05)**
	b)	Show that the parallelogram is rhombus ⇔ the diagonal are perpendicular to each other.	**(05)**
	c)	Find the orthogonal projection of $u = (1, -2, 3)$ along $v = (1,2,1)$ in R^3 with respect to Eclidean inner product space.	**(04)**
Q-5		**Attempt all questions**	**(14)**
	a)	Prove that any orthogonal set in inner product space V is linearly Independent.	**(05)**
	b)	Prove that x and y are orthogonal $\Leftrightarrow \|x + y\|^2 = \|x - y\|^2$	**(05)**
	c)	Find the Eigen values of matrix $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	let $T: V \rightarrow V$ be a linear transformation then prove that the following statements are equivalent. (i) T is orthogonal (ii) $\|Tx\| = \|x\|$, $\forall x \in V$ (iii) T maps orthonormal basis to orthonormal basis	**(06)**
	b)	Solve the system of linear equations by using Cramer's rule $x + y = 0$, $y + z = 1$, $z + x = -1$.	**(05)**
	c)	Show that $det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$.	**(03)**
Q-7		**Attempt all questions**	**(14)**
	a)	If $V = R^2$ and $x = (x_1, x_2)$ & $y = (y_1, y_2) \in R^2$ then show that the area parallelogram spread by x and y is $det(x, y)$.	**(05)**

b) Let V be an inner product space, then the map $f(x,y) =< x, y >$ is bilinear map. **(05)**

c) Prove that $\det(AB) = \det(A)\det(B)$. **(04)**

Q-8 **Attempt all questions** **(14)**

a) State and prove Riesz representation theorem. **(07)**

b) Apply Gram-Schmidth process to obtain orthonormal set$S = \{u_1, u_2, u_3\}$ Where $u_1 = (-1,0,1)$, $u_2 = (1,-1,0)$, $u_3 = (0,0,1)$. **(07)**